Linear and quadratic models of the southern Murray-Darling basin

Nigel Hall[1]
Canberra, Australia

Linear programming, a well-established technique for modelling agricultural and other systems has been extensively used to model irrigation systems in Australia. The models constructed have been used to assess the effects of changing water charges on farm incomes, water use and the effects of trade in water allocations. A linear programming model that reflects actual farm behaviour requires a large amount of data. Moreover, demand and supply functions estimated with linear programming models are necessarily stepped rather than smooth. Howitt has suggested a way of escape from these problems of linear programming models. His positive programming approach replaces a complex linear programming model with a much simpler quadratic programming model. In this paper, Howitt's approach is used to develop a quadratic programming model from an existing linear programming model of irrigated agriculture in the southern Murray-Darling basin of Australia. A comparison of the quadratic and linear models indicates that the quadratic programming model is smaller and simpler to specify and that it produces similar results to the linear model, in terms of cropping, trade and demand for irrigation water.

1. INTRODUCTION

Linear programming is a well-established technique for modelling agricultural and other systems. It has a number of advantages including the capacity to represent a major change in the way a system operates. In addition, the components of linear programming models can be broken down into manageable segments that can be discussed and checked with industry experts

The disadvantages of linear programming are the mass of data needed and the difficulty of validating the modelled results. Unlike an econometric model, there is no simple measure of goodness of fit for a mathematical programming model of a system.

Normally a mathematical programming model shows a stepped response to changes in for example, output prices. The more the number of binding constraints, the closer the model's response is to a smooth curve. Where the model represents aggregated systems of many paddocks or farms the changes can be expected to occur at different points for different farms. As a result the aggregate system will change gradually rather than at a single price. The steps in a response curve will thus be reduced by increasing the number of binding contraints and by disaggregating the matrix but only at the cost of increasing the complexity and data needs of the model.

Howitt, who suggested a method of replacing a complex linear programming model with a much simpler quadratic programming model, offers an escape from this dilemma. His quadratic model is easier to set up and necessarily produces smooth response surfaces without a high level of disaggregation [Howitt 1995; Howitt 1995b].

A related approach has been used by ABARE to derive supply responses from farm economic survey data on the basis of assumptions about profit maximisation by farmers and an assumed general form of the supply curve for each commodity [Kokic, Beare, Topp and Tulpule 1993]. This approach has been extended into a mathematical programming framework for analysis of the relationships between agriculture and

[1] Dr Nigel Hall is an economic consultant specialising in natural resource issues.

greenhouse gas emissions [Hall, Howden and Phipps1995], [Abdalla and Hall 1999].

1.1 The Positive Mathematical Programming approach

The linear programming approach assumes that unit costs are constant for any given activity. That is, marginal cost equals average cost. 'Corner solutions', where the model has only one single activity are common, unless the choice of activities is restricted by rotational and other constraints.

The Positive Mathematical Programming approach, in contrast to linear programming, assumes that unit costs (or sometimes other controlling factors) are an increasing function of the level of the activity. Thus, increasing the production of an activity also increase its unit cost. The proportions of multiple activities, such as two crops on a single farm are therefore controlled by the ratios of marginal costs to marginal revenues according to basic economic theory. In contrast a linear model has to assume rotational rules to achieve the observed pattern of cropping.

The approach addresses the common situation where the linear constraint set yields fewer binding constraints than observed activities. For example, a model with land water and labour constraints but a dozen planted crops in the region. Positive mathematical programming uses some of the linear programming dual values to respecify the constant linear costs in the objective function with increasing quadratic costs as a function of area cropped. This allows for more activities in the solution than there are binding constraints and a solution that calibrates exactly to the base year cropping.

The method is operated as follows. First run the linear programming solution with added calibration constraints to compel the desired cropping pattern. Next the dual values from the constrained LP are used to calculate the coefficients of the cost functions. This is straightforward algebra, on the assumption that the total cost curve is quadratic with no linear term. Finally the new cost coefficients are incorporated into the model and it is solved as a quadratic programming problem. The solution calibrates exactly with the observed outputs, has no additional constraints, and has the same objective value and the same dual values on resource constraints as the constrained linear programming model. The estimation of the coefficents is presented in Appendix 1.

1.2 General description of the IMMS model

This paper originated in a request to update an existing linear programming model of the irrigation system in the southern Murray-Darling basin of Australia. The new linear model was used to develop a simpler positive mathematical programming (quadratic) model so that the two approaches can be compared using closely related models based on the same input data. In this paper the two models are compared in terms of matrix complexity, how closely they represent the current situation and water demand functions at system and regional level.

The IMMS (Integrated Murray-Murrumbidgee Modelling System) model was developed by ABARE in 1993 for the Murray-Darling Basin Commission. The State Government Departments responsible for agriculture and water management supplied the base data on water use and irrigation practices and the Murray-Darling Basin Commission contributed its experience of modelling the river system [Hall, Poulter et al. 1994]. The model was used in development of the COAG water reforms [AATSE 1999], [COAG 1994], These reforms affirmed the need for water trading and full cost recovery from irrigation pricing as fundamental to efficient water use in Australia. The model has assisted policy makers by providing estimates of volumes and prices of trade water, gains from trade and losses in farm income from higher irrigation water charges implied by full cost recovery.

The model is a spatial equilibrium model designed to represent the main irrigation areas and river pumpers of the southern Murray-Darling basin. The objective is to simulate the competitive market equilibrium. This is done by maximising the domestic and foreign consumer valuation of crop and livestock products produced in the irrigation regions and in the rest of Australia less the costs of variable inputs in the irrigation regions and the rest of Australia. Each irrigation region is modelled using linear programmming.

A model of the river system and a model of product supply and demand link the region models. Water trading, changes in water use in each region, their effect on the salinity of the Murray River and the cost of this salinity to the economy are modelled (Hall, Poulter and Curtotti 1994).

The products included in the model are: rice, grapes; citrus, apples and stone fruit, wheat (representing winter crops), summer crops (soybeans and canola), vegetables (processing tomatoes) and cotton. Livestock products include wool, prime lamb, beef and milk. Livestock feed comes from annual and perennial pastures. Lucerne hay is produced as a cash crop and concentrated feeds for dairy cattle are accounted for in cash costs. The irrigation areas included in the model contain about half of the irrigation farming in Australia.

The water use constraint is defined for each region based on the current requirements of each crop. Crops and pasture can be irrigated or grown without irrigation in regions where this is feasible. Pastures and Lucerne have three possible water application levels; full, reduced and dryland with appropriate costs and yields. Wheat can be grown under irrigation or as a dryland crop. Other than wheat, field crops, cotton, vegetables and tree and vine crops are assumed to have only one possible water application rate.

Water flow in the river system is represented on the basis of four seasons. Water accessions, use and losses are progressively accounted for as the water flows to the sea. The period of the model is a full year for everything except water flows and water use. There are likely to be advantages in disaggregating the period to represent seasonality in the use of irrigation water.

The model is highly aggregated and does not account for farm capital, regional capital, including the drainage system, the feedback in the environmental system, off-farm systems and services, long term trends, risk and uncertainty. The model was updated in 1998 by the author for ABARE.

2 COMPARISON OF LINEAR AND QUADRATIC MODELS

The models are compared on three criteria, complexity, closeness to actual land use, water trading and demand response for water at basin and region level. There are 20 regions in the model but discussion is focussed on the whole basin and some key crops bcause a full comparison of 20 regions by 20 crops would be too bulky.

In terms of matrix size the linear model has 1516 rows and 4159 columns with 17592 non-zero cells. The quadratic programming model has only 499 rows and 3673 columns with 14705 non-zero cells. Overall matrix size is reduced to 29 per cent of the linear model although the reduction in non-zero cells is much less, 83 per cent of the linear model. Both models have the same matrix of water flows and trading allowing trade between all 20 regions, between seasons and keeping track of water flows through the model.

The main areas removed from the model concern rotations and constraints on crop growing including soil types. These are some of the least certain data entered into the model because of the wide variety of farm practice and uncertainties about the degree to which crop performance varies between soils as defined. For example, it was usual to plant tree and vine crops only on sandy or tile drained soils but vines are now being grown successfully on heavy clays formerly used for rice in the MIA.

TABLE 1. AREAS IRRIGATED

	Linear Model '000 ha	Quadratic Model '000 ha	Actual 1996-97 '000 ha
Rice	166	162	162
Wheat	1319	1046	1063
Canola	90	29	29
Soybean	17	3	3
Lucerne	38	32	32
Annual pasture	729	1379	1376
Perennial pasture	585	293	295
Cotton	27.5	17	17
Vegetables	32.9	28.7	28.8
Grapes	51.4	46.9	46.9
Citrus	29.1	32.1	32.1
Stone fruit	10.6	11.7	11.7
Pome fruit	7.3	6.9	6.9
All Irrigation	3102.8	3087.3	3103.4

Table 1 allows a comparison of the total areas irrigated in the quadratic programming model, the linear programming model and actual areas irrigated in 1996-97. The quadratic model is a generally close fit to the actual crop areas. The linear model is a much poorer fit to the actual areas even though considerable effort was put into achieving a good fit. There is too much wheat, canola, soybeans, vegetables and perennial pasture and too little annual pasture

A second comparison of the two models is of their water use and trading performance. Water use and trading are a key focus of the modelling system. The two models are compared in Table 2.

The two models are similar in total water use although the volume of trade for the quadratic model is only half that for the linear model. The average difference in water use for individual regions is 12 per cent with slightly more water being used by the linear model. The direction of trade from the two models is the same in 70 percent of regions. The greatest difference is the large sale by river diverters on the eastern Murray

River in New South Wales in the linear model. 125Gl of water are sold from this region, in the linear model, while none is sold by the quadratic version. The region has an excess of water allocation compared to its use and so can sell a large volume without greatly changing its own use. However in the abundant water situation modelled, the price of water is close to zero in trade so that the decision to sell or not sell water is not very critical. In a more normal year when water is scarce the results could be different.

. TABLE 2. WATER USE AND TRADE

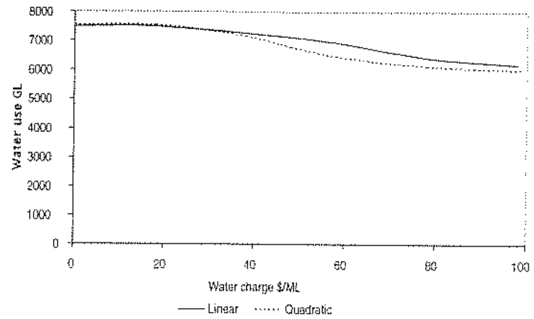
	Water use Linear GL	Quad GL	Net trade Linear GL	Quad GL
MIAIRRIGN	1162	1029	0	40
CIAIRRIGN	524	505	0	0
MBDIVRTER	365	347	28	111
MILTDEAST	1072	938	-37	0
MILTDWEST	392	381	0	0
MUENSWDIV	277	316	226	0
MUWNSWDIV	118	109	-10	0
LODARLING	264	203	-189	-126
SHEPPARTN	288	271	-28	-15
GOULBURN	657	639	-67	-56
MURVALLEY	458	395	-52	10
ROCHESTER	368	341	-25	2
TORRUMBRY	689	640	-81	-40
PYRAMIDHL	288	353	62	7
MUEVICDIV	101	72	19	0
GOUVICDIV	93	88	57	62
MUWVICDI	51	65	32	0
SUNRAYSIA	149	152	9	5
RIVERLAND	337	370	55	0
LOWMURRAY	120	147	0	0
TOTAL	7771	7361	488	237

Regions are defined in Appendix 2.

A further comparison of the two models is their responsiveness to levels of water charges. An increase in water charge can be expected to lead to a reduction in water use for irrigation as the cost of applying water increases. The models were used to simulate this response over the range of water charges from the current level to approximately $100 per ML.

Current levels of water charges by region average about $20 per ML ranging from under $7 per ML for river diverters (who also pay their pumping cost) to over $50 per ML on pumped and pressurised systems in South Australia. For the simulation $20 per ML was added to the current charge at each round until the average charge was near $100 per ML. This charge is far above any that is likely although the possible inclusion of interest charges on the water storage infrastructure of the industry after 2000 will lead to significant increases in water costs to the industry. The response of the two models is shown in the figure.

FIGURE 1 WATER DEMAND: TWO MODELS



At the lower end of the scale below $40 per ML, there is little difference in the two models' responses. Beyond this charge level the two lines diverge a little. Both slope downward to the right indicating that increased charges reduce water use but that the change is not very large. The elasticity of demand for water is a quantitative measure of the relationship between water use and water charges, this can be calculated from the data. The linear model has an elasticity of –0.11 while the quadratic model suggests an elasticity of –0.14. Both are quite inelastic implying that large changes in charge are needed to alter water use. This is not surprising because these are short-term models and big changes in water use may require long term investment. Moreover, water charges are minor for most enterprises with the possible exception of rice. These elasticities are much less than those quoted by Hall, Poulter and Curtotti [1994] (-0.99) but this is because they cover only a small range. When the range is increased to a level needed to force water use down to zero the elasticities are estimated at –1.15 and –1.26 respectively.

3. DISCUSSION

Two models of the same system and using the same input data are compared in this paper. One is based on linear programming; the other is a positive mathematical programming model. The difference relates to the parts of the models that describe the areas planted of crops and the rotational relationships between them. The agronomic and regional data and the water flows in the models are identical. This allows a comparison of two approaches to modelling regional land use. The linear programming approach is long established and has been shown to give a good simulation of actual land use if sufficient detail is included (see for example [Greiner and Hall 1997]. The quadratic programming approach is easier to implement and involves a smaller and simpler matrix because it accepts the actual areas of crops, as implicitly containing all the data needed to model crop area. However, it may seem a less reliable approach because modellers do not use all the detailed knowledge of agronomy that is traditionally at the heart of linear programming modelling.

The comparisons made in this paper showed that the quadratic programming model gave a better reproduction of areas cropped than the linear programming model. More extensive modelling with additional constraints would improve the fit of the linear model at a cost in matrix size and researcher time but the current linear model is inferior, in this regard, to the quadratic model.

The second comparison is of the regional water use in the model and the estimated trade between regions. In simulating a year of very abundant water for irrigation, full water allocations were assumed available in New South Wales and South Australia, and a traditional level of sales water was available in Victoria. Under these circumstances the estimated trading price for water allocations was close to zero and the linear programming model traded much more water. Trade in most regions was however in the same direction in both models with only one region showing serious difference in water trading. This region had a surplus of water allocations over use in the base situation and this "sleeper" allocation accounts for the extra trade.

The actual volumes of trade currently occurring are much less than those modelled. There is great variation between years with much greater sales in dry years and a general upward trend in sales. In 1996-97 total water traded between the regions used in the model was about 100 GL. All models including this model and the Victorian DNRE model [Agriculture and DNRE 1998] estimate an increase in trade with opening of interstate trade but the extent of this varies between simulations. The linear model estimates trade as almost 500 GL while the quadratic programming model estimates trade as just over 200 GL.

The third comparison made is of the responsiveness of water quantity demanded to the cost of water to irrigators. This indicated that both models had an inelastic demand response with small changes in volume used for large changes in water charge. The quadratic model had more elastic demand. This reflects particularly the response of rice growers. In the linear model there is a no reduction in rice growing until the water charge reaches $80 per ML after which there is a marked reduction. In contrast the quadratic programming model shows a more gradual reduction beginning at a lower charge. This difference accounts for the more elastic demand function for the quadratic programming model as well as some of the differences in trading response reported.

The comparisons show that the two models are broadly equivalent and that neither can be said to be a superior model on every score. However, the quadratic programming model is better at reproducing the current cropping pattern, estimates lower levels of trade nearer to current trading and is more realistic in its assumption that farmers will change crops gradually rather than in unison at a particular price.

APPENDIX 1. QUADRATIC COST CURVES FROM IMMS

Suppose two crops wheat and oats with a land area constraint for one type of land. An LP specification will give either one crop or the other over the whole area. Assume that wheat has a quadratic cost function and that the actual land use is one-third oats to two-thirds wheat. Call these areas Xw and Xo. The model is,

$\text{Max } J = PoXo - CoXo + PwXw - \alpha wXw - 0.5 * \gamma wXw^2$

Subject to $Xo + Xw < A$

Where J is profit, Po and Pw are prices of oats and wheat, Co is a linear cost per ha of oats and α and γ are linear and quadratic cost coefficients for wheat. Oats is left linear to simplify the exposition. At the point where the areas coincide with the base situation, the marginal profit of oats must equal the marginal profit of wheat. If we assume a linear specification and constrain the areas to fit the base situation then the dual of constraining wheat area will be the opportunity cost of keeping wheat area down to the base level (assuming wheat is more profitable).

The problem is to solve for the values of the linear and quadratic coefficients given the dual values on the constrained areas.

$\lambda 2w = MCw - ACw$

The dual value of wheat area is the marginal cost of wheat less the average cost

$Cw = Acw$

Linear wheat cost coefficient is equal to average cost of wheat growing

From the positive mathematical programming objective function J,

$MCw = \alpha w + \gamma wXw$ and $Acw = \alpha w + 1/2\ \gamma wXw$

$\lambda 2w = \alpha w + \gamma Xw - \alpha w - 1/2\gamma Xw = 1/2\gamma wXw$

$\underline{\gamma w = 2\lambda / Xw}$

$Cw = \alpha w + 1/2\gamma Xw$

$\alpha w = Cw - 1/2\gamma Xw$

$\underline{\alpha w = Cw - \lambda}$

At the base are off wheat and oats the marginal net revenue form wheat equals the net revenue from oats equals the opportunity cost of land.

The coefficients are calculated within the IMMS framework to allow recalculating of coefficients or changes to be easily and consistently made.

APPENDIX 2 REGIONS

MIAIRRIGN	NSW	Mirrool, Yanco, Tabbita, Benerembah, Wah Wah
CIAIRRIGN	NSW	Coleambally
MBDIVRTER	NSW	Diverters in Murrumbidgee Valley
MILTDEAST	NSW	Berriquin and Denmien
MILTDWEST	NSW	Wakool, Tullakool & Deniboota
MUENSWDIV	NSW	East of Murrumbidgee Mouth
MUWNSWDIV	NSW	Irrigation West of Murrumbidgee
LODARLING	NSW	Lower Darling South of Menindee Lakes
SHEPPARTN	Vic.	Shepparton
GOULBURNC	Vic.	Central Goulburn
MURVALLEY	Vic.	Murray Valley
ROCHESTER	Vic.	Rochester/Campaspe
TORRUMBRY	Vic.	Torrumbarry
PYRAMIDHL	Vic.	Pyramid / Boort
MUEVICDIV	Vic.	Murray diverters in Vic. East
GOUVICDIV	Vic.	Goulburn diverters in Vic.
MUWVICDIV	Vic.	Murray diverters in Vic. West
SUNRAYSIA	Vic.	Mildura, Merbein, Red Cliffs, Robinvale
RIVERLAND	SA	Riverland Irrigation Areas
LOWMURRAY	SA	Lower Murray below Morgan

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